Name of School	Terrington St Clement Community School
Policy review Date	September 2022
Date of next Review	September 2024
Who reviewed this policy?	Frankie Penfold

Calculation Policy

Progression towards a standard written method of calculation

Introduction

This calculation policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014). It provides guidance on appropriate calculation methods and progression. The content is set out in yearly blocks under the following headings: addition, subtraction, multiplication and division.

Statements taken directly from the programmes of study are listed in bold at the beginning of each section.

Children will use mental methods and jottings as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence.

Aims of the Policy

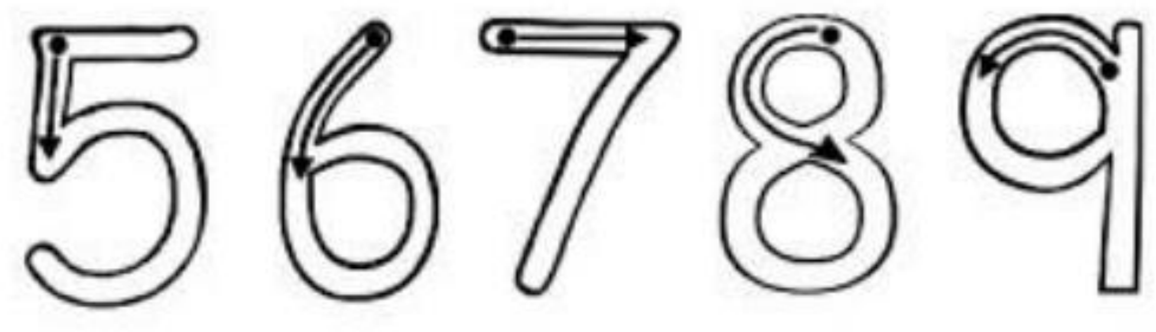
- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations
- To ensure that children can use these methods accurately with confidence and understanding

How to Use This Policy

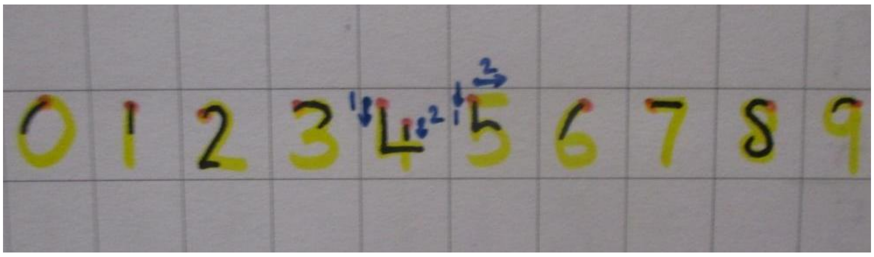
- Use the policy as the basis of your planning but ensure you use previous or following years' guidance to allow for personalised learning
- Always use Assessment for Learning to identify suitable next steps in calculation for groups of children
- If, at any time, children are making significant errors, return to the previous stage in calculation
- Cross reference with the mental maths policy for guidance on key facts, key vocabulary and mental methods
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate
- Encourage children to make sensible choices about the methods they use when solving problems
- Consider how you build each of these skills using the Concrete/Pictorial/Abstract approach that the school has adopted (CPA)

Digit Formation

In Maths, digit formation is just as important as letter formation in writing. Below are examples of how we teach our children to write numbers. Using any other notation can be perceived as a misconception. It's really important all children are using these notations.



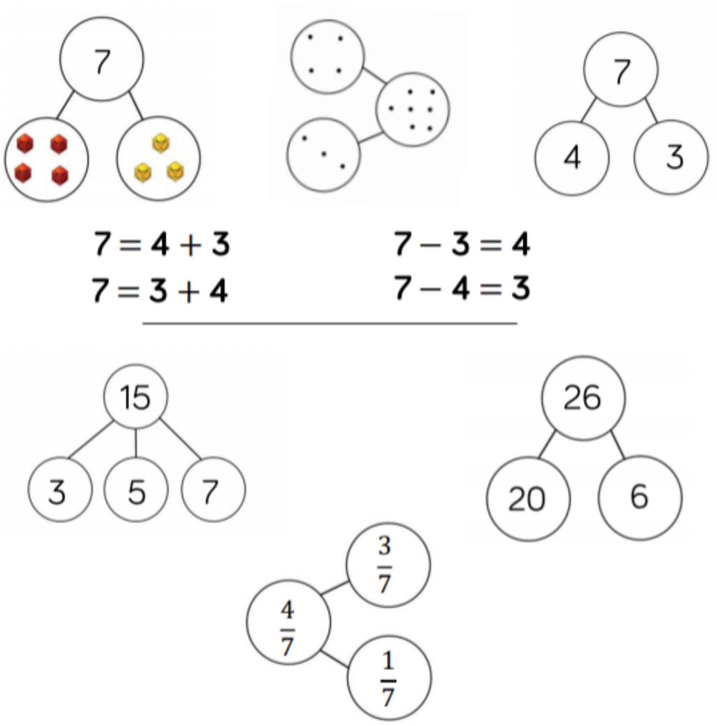
We teach the children to record numbers in maths using one digit per box and model this as below:



Visual Representation/Concrete Materials - Addition & Subtraction

It is essential that we introduce children to a common set of visual representations. The following strategies will be taught and built upon each year as appropriate. *(NB This section is taken directly from White Rose Maths Hub Resources)*

Part/Whole Model



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

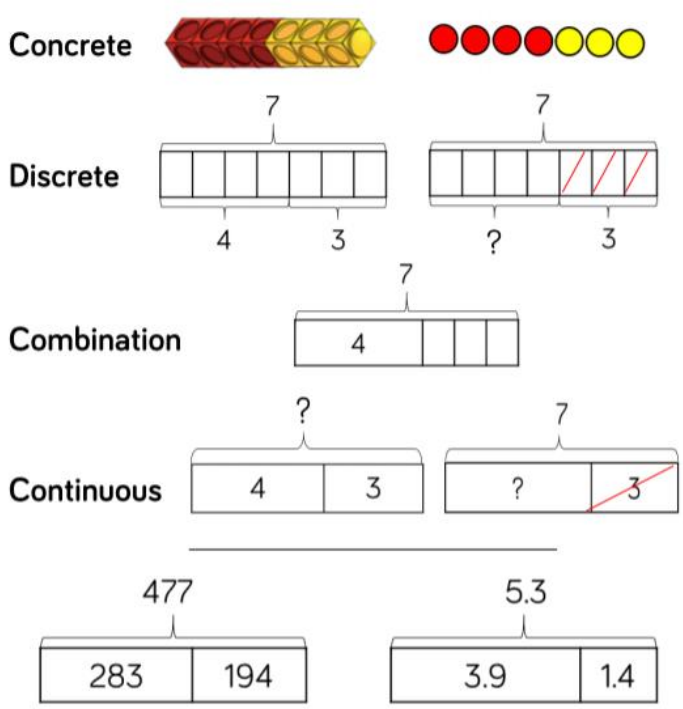
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (Single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

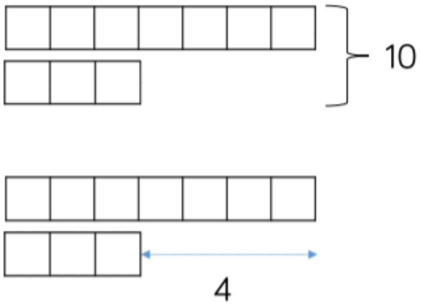
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

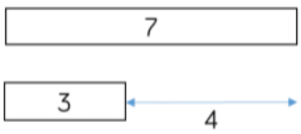
Discrete



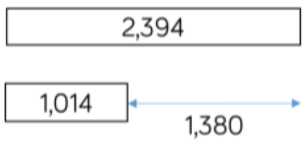
$7 + 3 = 10$

$7 - 3 = 4$

Continuous



$7 - 3 = 4$



$2,394 - 1,014 = 1,380$

Benefits

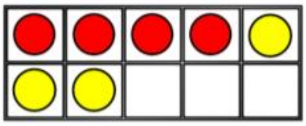
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

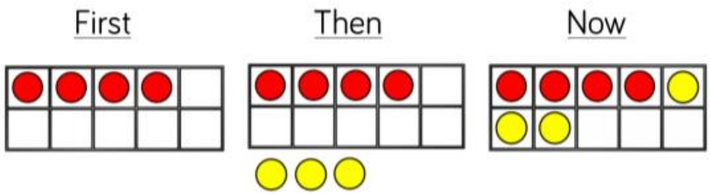
When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Tens Frame (within 10)

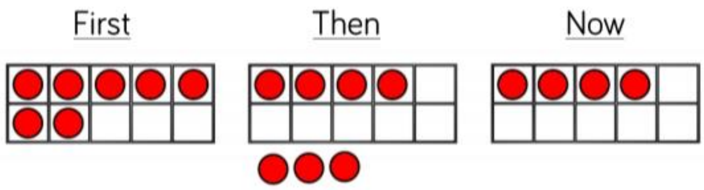


$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

4 is a part.
3 is a part.
7 is the whole.



$4 + 3 = 7$



$7 - 3 = 4$

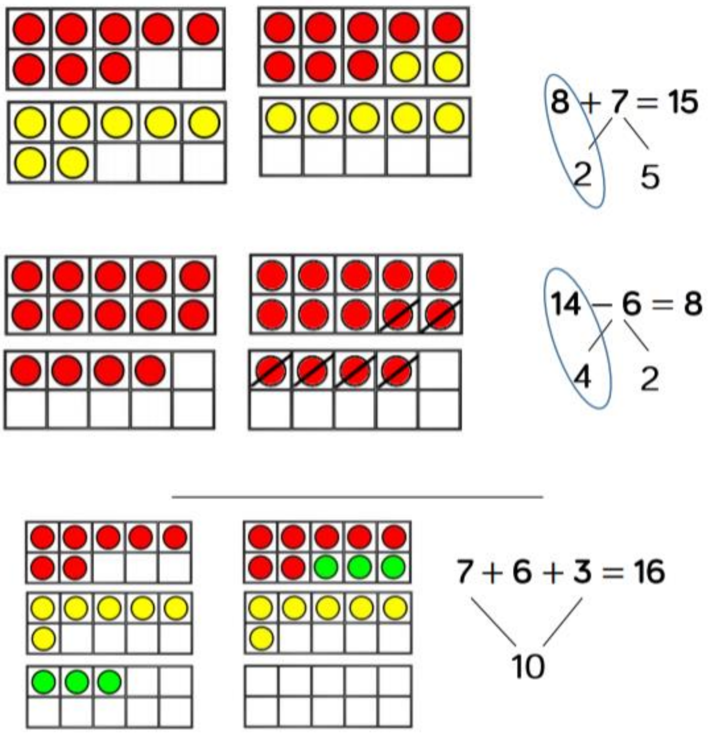
Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Tens Frame (within 20)



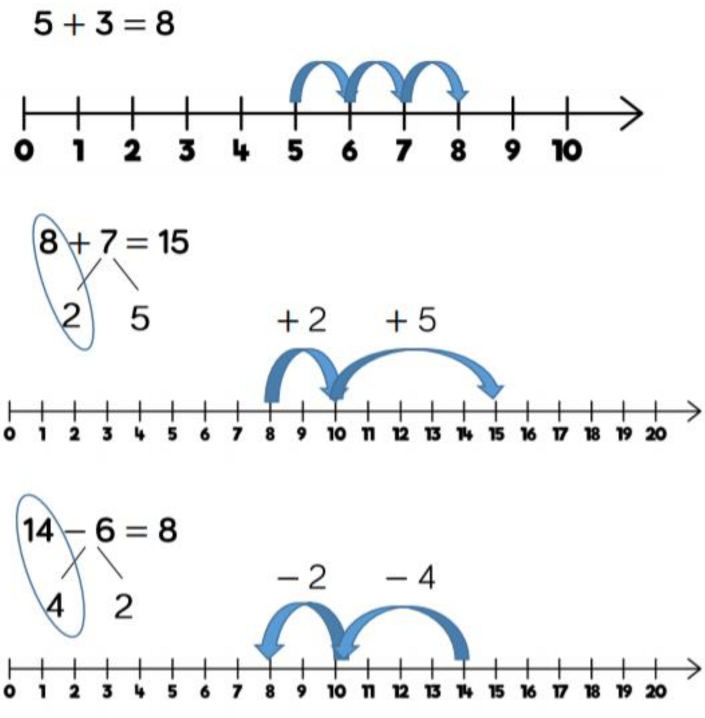
Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Number Lines (Labelled)



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

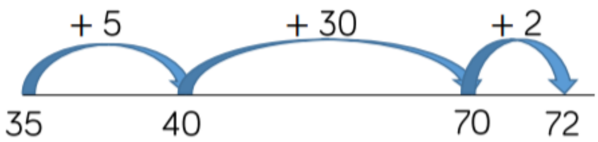
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

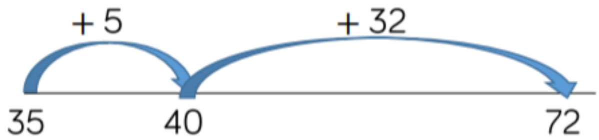
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (Blank)

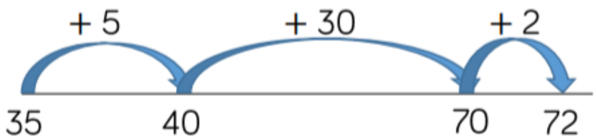
$35 + 37 = 72$



$35 + 37 = 72$



$72 - 35 = 37$



Benefits

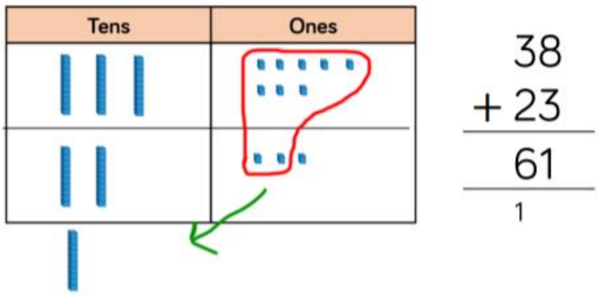
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

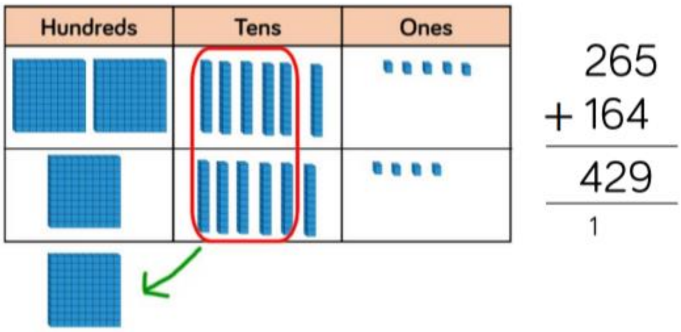
Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Base 10/Dienes (Addition)





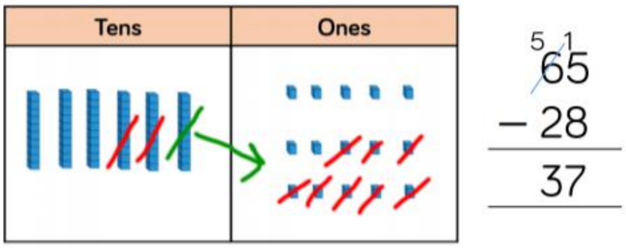
Benefits

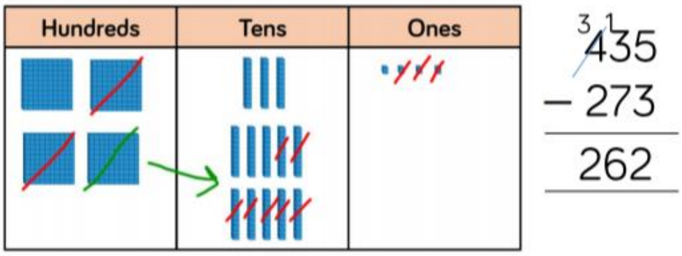
Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

Base 10/Dienes (Subtraction)



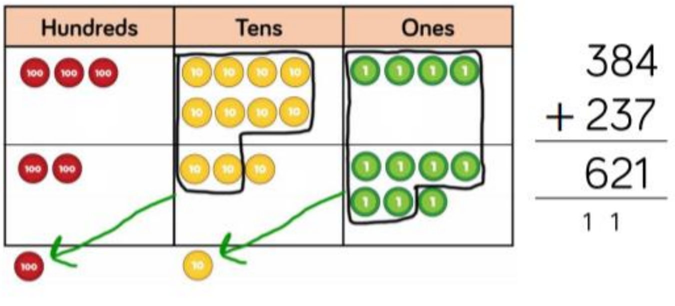


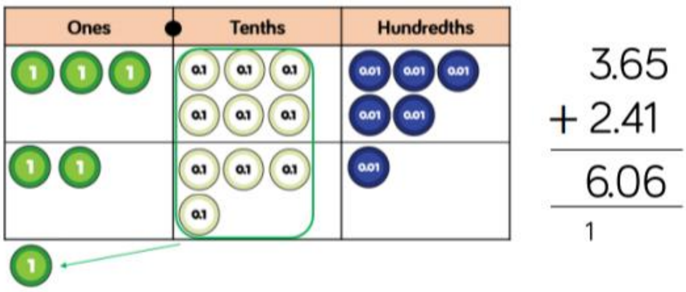
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.
This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (Addition)





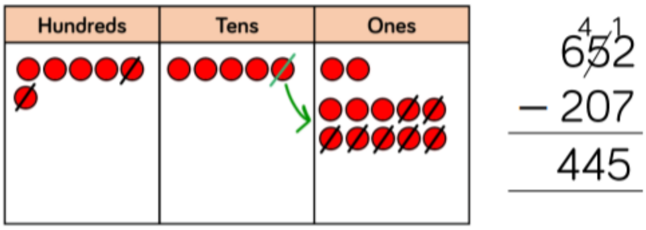
Benefits

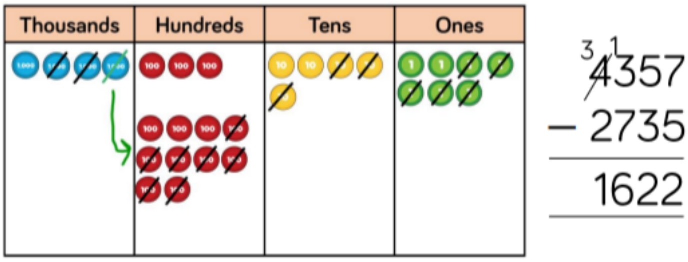
Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)





Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

Stages in Addition

Addition - Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. They will begin to relate addition to combining two groups of objects, first by counting all and then by counting on from the largest number.

They will find one more than a given number.

In practical activities and through discussion they will begin to use the vocabulary involved in addition.

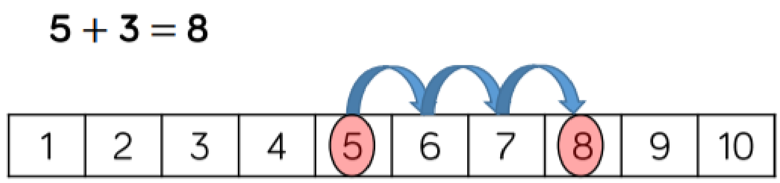
'You have five apples and I have three apples. How many apples altogether?'

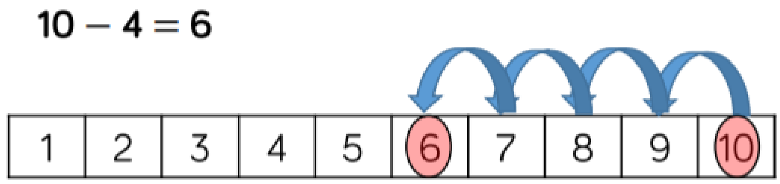
Addition - Year One

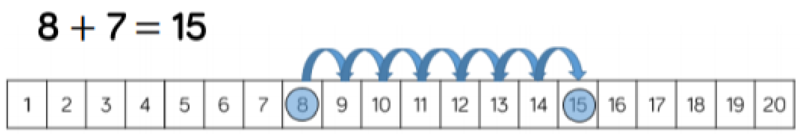
- Given a number, identify one more
- Use number bonds within 20
- Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign
- Add one- digit and two-digit numbers within 20, including zero
- Solve missing number problems eg 10 + ☐ = 16

Children will continue to practise counting on from any number e.g. 'Put five in your head and count on four.'

Initially use a number track to count on for addition, counting on from the largest number:







Then move onto a marked number line (see visual representation section above).

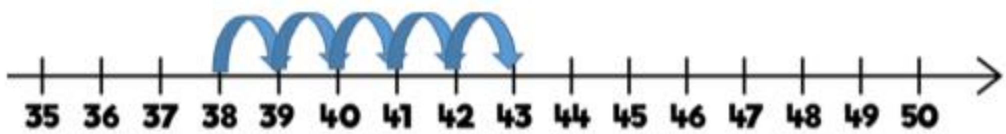
Ensure children are confident with using a marked number line before moving on to an empty number line (see year two guidance).

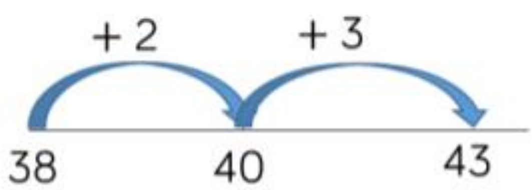
Continue to practise counting on from the largest number for addition with totals within 20.

<u>Addition - Year Two</u>

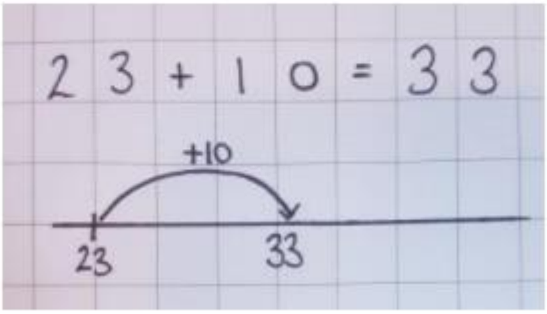
- Add numbers using concrete objects, pictorial representations, and mentally, including:
  - A two digit number and ones
  - A two digit number and ten
  - Two two-digit numbers
  - Three one-digit numbers

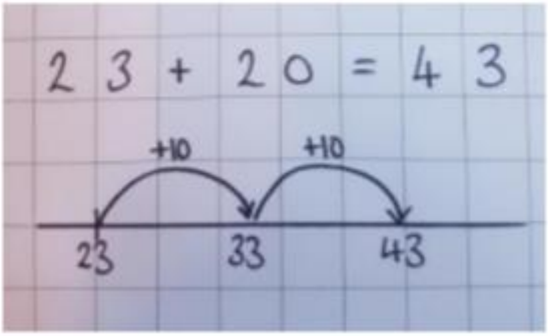
Counting on in ones using an number line, progressing to an empty number line within 100:





…and in tens

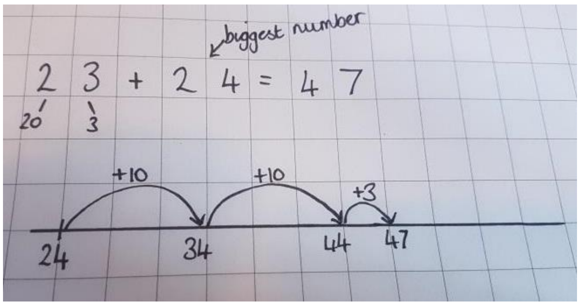




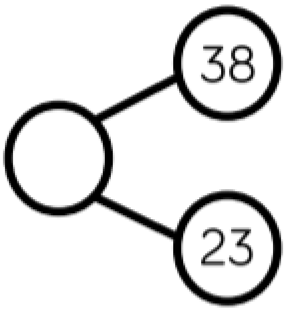
Use in conjunction with a 100 square to show jumps of tens.

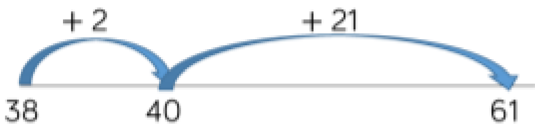
23 + 24 = 84

‘Put the biggest number first, and then partition the smaller number (23 = 20 + 3) and count on: 24 + 20 + 3.’



Once confident with this approach, consider other ways to partition the number to be added.





Use in conjunction with a 100 square to show jumps of tens and ones/units.

Also use the partitioning method to add two two-digit numbers:

43 + 25 = 68

40 3 20 5

40 + 20 = 60
3 + 5 = 8

60 + 8 = 68

‘Partition the numbers into tens and ones/units.
Add the tens together and then add the ones/units together. Recombine to give the answer’.

Then move on to calculations that bridge the tens:

48 + 36 = 40 + 8 + 30 + 6

40 + 30 = 70
8 + 6 = 14

70 + 14 = 84

48 + 36 = 84

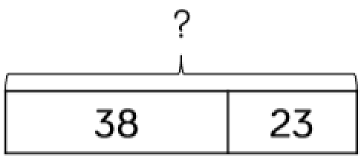
This is an alternative way of recording the partitioning method.

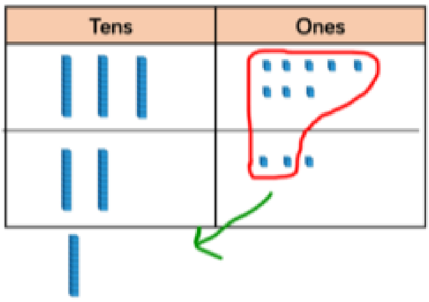
Further develop addition with numbers that bridge 100, using a 200 grid to support.

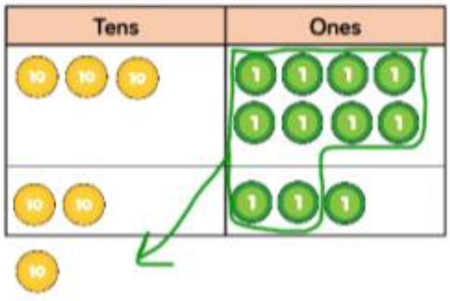
Addition - Year Three

- Add mentally, including:
  - A 3 digit number and ones
  - A 3 digit number and tens
  - A 3 digit number and hundreds
- Add numbers with up to three digits, using formal written method of columnar addition

Start to use formal column addition alongside visual representations:







Introduce the expanded written method with the calculation presented both horizontally and vertically (in columns). Initially use calculations where it has not been necessary to bridge across the tens or hundreds:

63 + 32 = 95

60 + 3
+ <u>30 + 2</u>
90 + 5 = 95

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then…

6 3
\+ 3 2
\+ 5 (3 + 2)
9 0 (60 + 30)
9 5

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

This will lead into the formal written method…

6 3
\+ 3 2
9 5

Use the language of place value to ensure understanding: 'Three add two equals five. Write five in the units column. 60 add 30 equals 90. Write 9 (90) in the tens column.

NB Informal/mental methods would be more appropriate for numbers of this size, but use two-digit numbers when introducing the columnar method.

Then introduce calculations where it is necessary to bridge, returning to an expanded method initially:

68 + 24 = 92

60 + 8
\+ 20 + 4
80 +12 = 92

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then…

6 8
\+ *2 4*
\+ *1 2* *(8 + 4)*
8 0 *(60 + 20)*
9 2

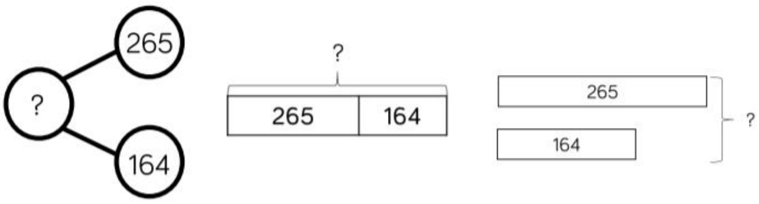
Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

If children are ready, introduce the formal written method, where it is necessary to 'carry' ten from the units to the tens column:

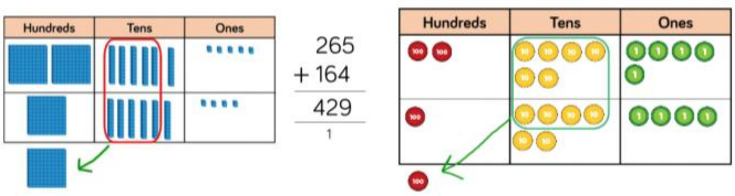
$$\begin{array}{r} 38 \\ +\,23 \\ \hline 61 \\ \hline {\scriptstyle 1}\ \end{array}$$

Use the language of place value to ensure understanding: 'Eight add three equals 11. Write one in the units column and 'carry' one (10) across into the tens column. 30 add 20 and the ten that we 'carried' equals 60. Write 6 (60) in the tens column. 61 is the answer.

Continue to make use of the empty number line to add a 3-digit number and ones, tens and hundreds, including those that bridge and represent in different ways to aid understanding. You may need to revert back to a more expanded method or column addition as above, when starting to work with larger numbers.



265 + 164 = 429

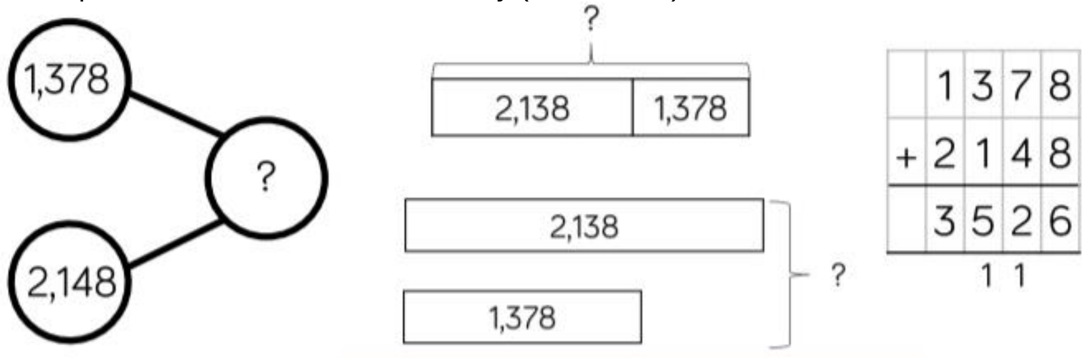


Addition - Year Four

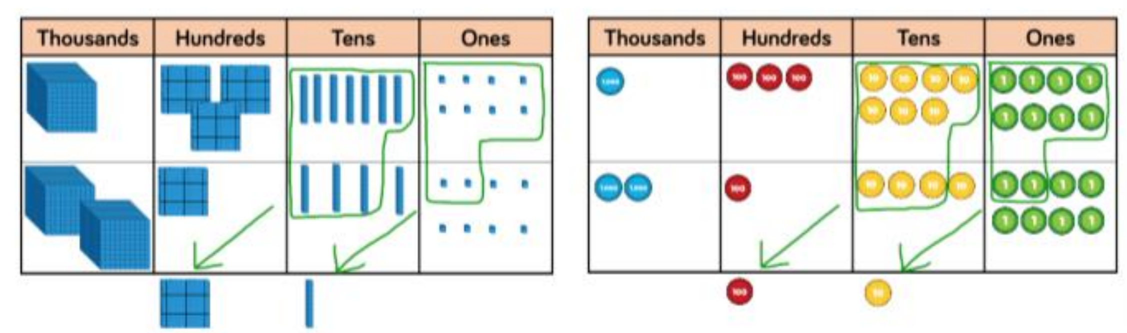
- Add numbers with up to 4 digits using the formal written method of columnar addition where appropriate

Continue to teach the use of empty number lines with three and four digit numbers, as appropriate.

Further develop the formal written method of addition, with three-digit and then four-digit numbers. Revisit the expanded method first, if necessary (see Year 3).



1,378 + 2,148 = 3,526



Move onto adding four-digit plus three-digit numbers. It is important that children understand the importance of position and place value. Encourage them to use 0 as a place holder if necessary:

1845 + 526 = 2371

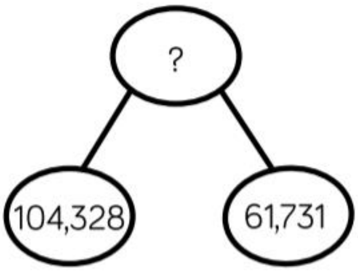
 1 8 4 5
+0 5 2 6

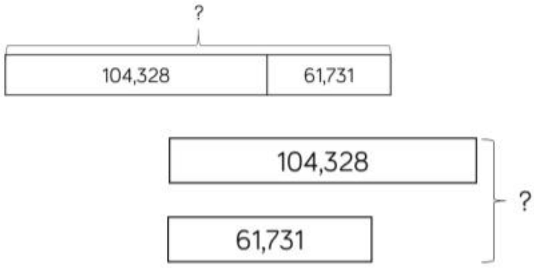
2 3 7 1
1   1

Continue to develop with addition of two four-digit numbers and with decimals (in the context of money or measures).

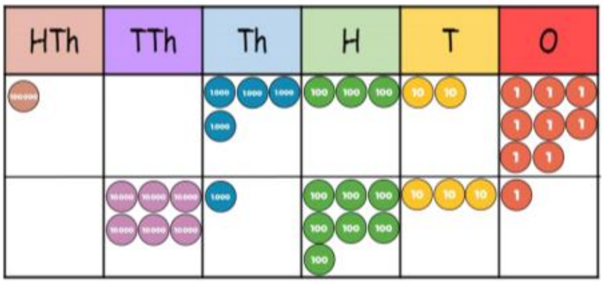
Addition - Year Five & Six

- Add whole numbers with more than 4 digits, including using formal written method (columnar addition) - *Year 5 objective, there are not separate objectives for Y6*



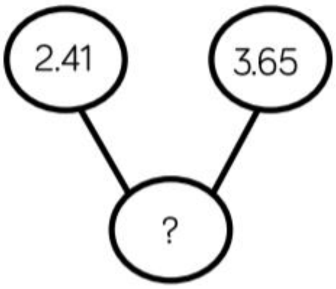


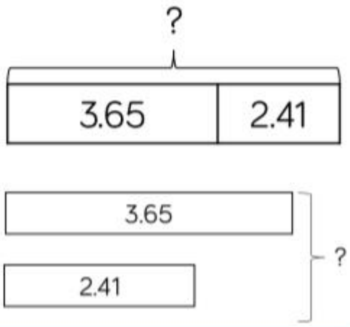
104,328 + 61,731 = 166,059



1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
		1			

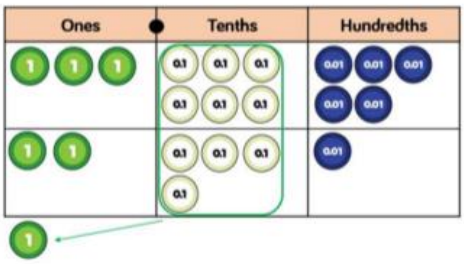
Ensure children are confident adding numbers including those with up to 3 decimal places:

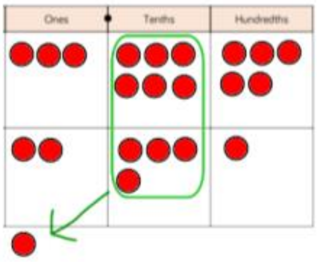





```
  3.65
+ 2.41
  6.06
  1
```

3.65 + 2.41 = 6.06





Teaching children to confidently use 0 as a place holder is essential when working with decimals as is ensuring that decimal places line up prior to calculating an answer.

Stages in Subtraction

Subtraction - EYFS

Children will engage in a variety of counting songs and rhymes and practical activities.

In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.

They will begin to relate subtraction to ‘taking away’ using objects to count ‘how many are left’ after some have been taken away.

6 – 2 = 4

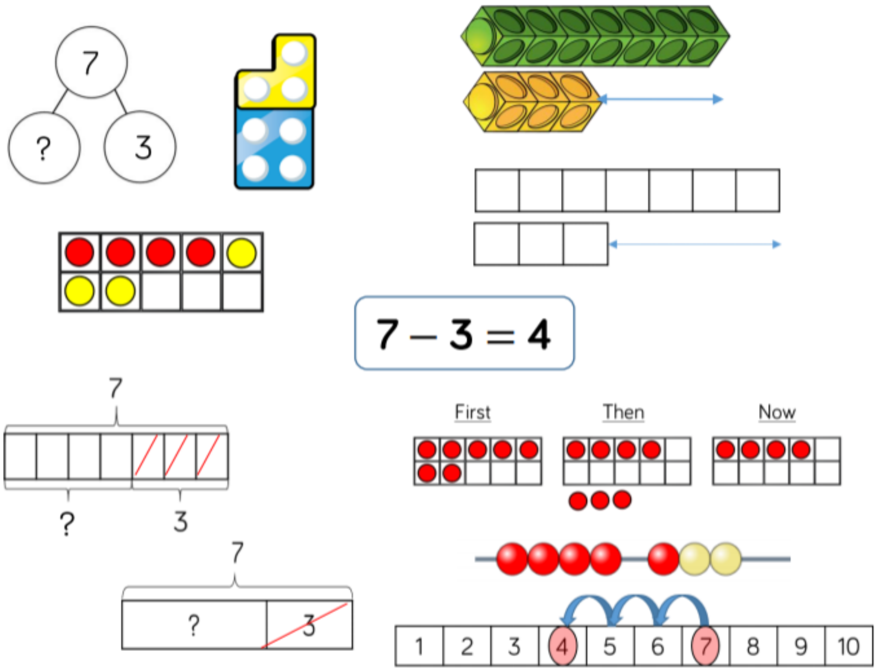
‘Take two apples away. How many are left?’

Children will begin to count back from a given number.

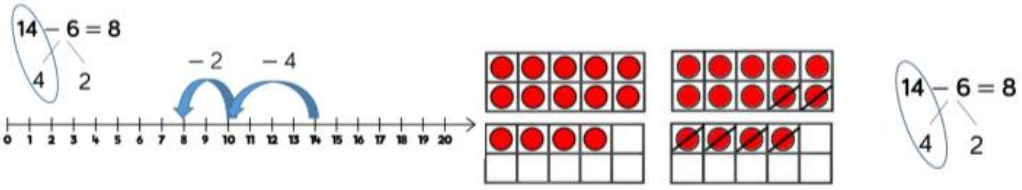
Subtraction - Year One

- Given a number, identify one less
- Use number bonds and related subtraction facts within 20
- Read, write and interpret mathematical statements involving subtraction (-) and the equals (=) sign
- Subtract one- digit and two-digit numbers within 20, including zero
- Solve missing number problems eg 15 = 20 - □

Children will continue to practise counting back from a given number. Initially use a number track to count back for subtraction alongside other visual representations.



This will then progress to using a marked number line alongside familiar visual representations. As they become more confident, they should be encouraged to find the number bond to subtract to 10 when partitioning the subtracted number. This will continue to be developed in Year 2.

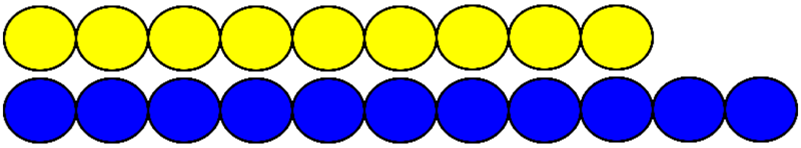


Counting on to find a small difference:

Introduce complementary addition to find differences (only use for small differences). The use of models is extremely important here to understand the idea of “difference”.

Count up from the smallest number to the largest to find the difference using resources, e.g. cubes, beads, number tracks/lines:

11 – 9 = 2

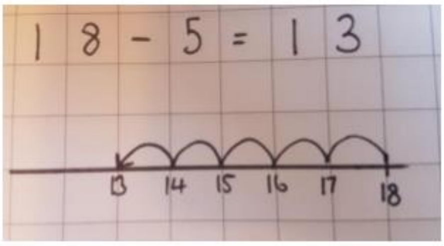


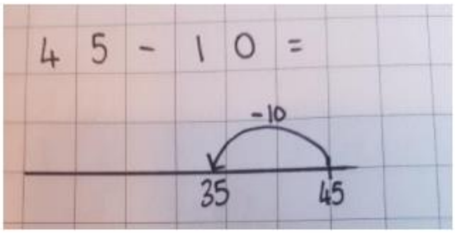
The difference between nine and eleven is two.

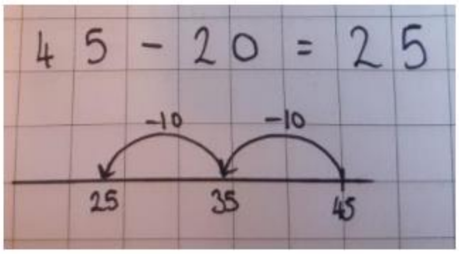
<u>Subtraction - Year Two</u>

- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
    - A two digit number and ones
    - A two digit number and tens
    - Two two-digit numbers

Count back on an empty number line in ones and then in tens:

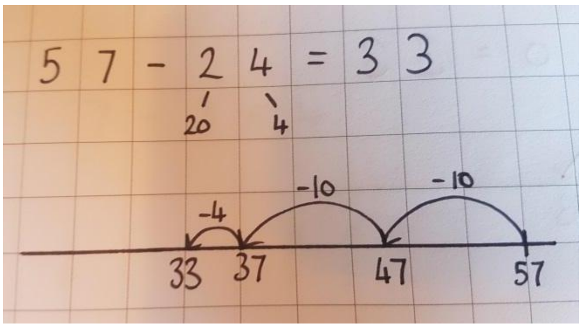




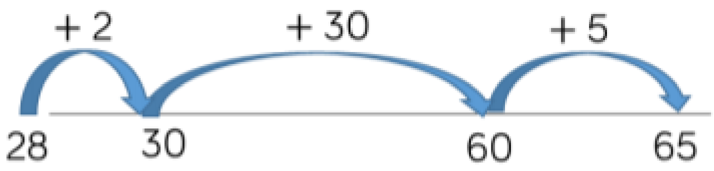


Use in conjunction with a 100 square to show jumps of tens.

Then move onto subtraction using partitioning on an empty number line.

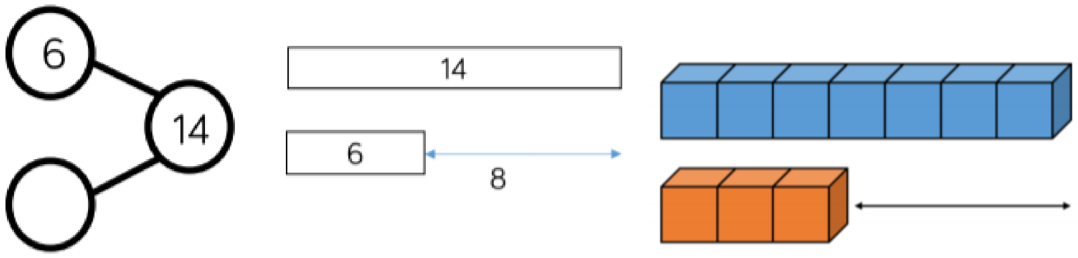


If children are confident with this method, consider different ways to partition the number being subtracted:



Counting on to find a small difference

Introduce complementary addition to find differences (only use for small differences). The use of models is extremely important here to understand the idea of "difference" (see Y1 guidance).

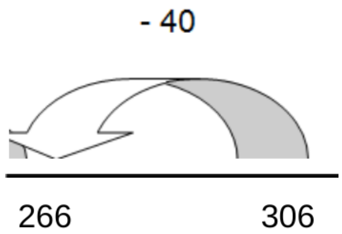


<u>Subtraction - Year Three</u>

- Subtract mentally, including:
  - a 3 digit number and ones
  - a 3 digit number and tens
  - a 3 digit number and hundreds
- Subtract numbers with up to three digits, using formal written method of columnar subtraction

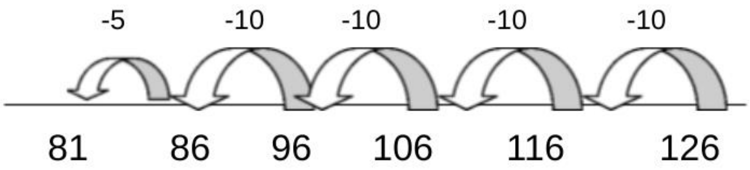
Continue to make use of the empty number line to subtract a 3-digit number and ones, tens and hundreds, including those that bridge:

306 - 40 =

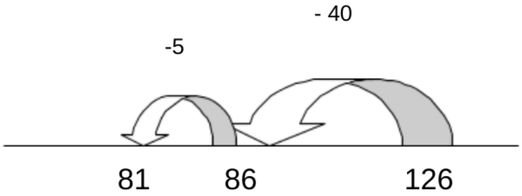


Further develop the use of the empty number line with calculations that bridge 100:

126 – 45 = 81



Then use more efficient jumps:



Introduce the expanded written method with the calculation presented both horizontally and vertically (in columns). Use two-digit numbers when introducing this method, initially:

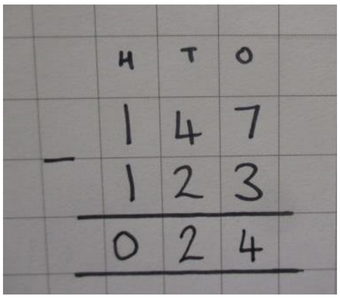
78 – 23 = 55

  70 + 8
−20 + 3
  50 + 5 = 55

'Partition numbers into tens and ones/units. Subtract the ones, and then subtract the tens. Recombine to give the answer.'

NB In this example decomposition (exchange) is not required.

You might replace the + sign with the word 'and' to avoid confusion.
This will lead into the formal written method (initially without any exchanging):



Introduce the expanded written method where exchange/decomposition is required alongside visual representations:

73 − 27 = 46

  70 + 3   becomes   60 +13
- 20 + 7   - 20 + 7
                       40 + 6 = 46

73 is partitioned into 60+13 in order to calculate 73-27

When children are confident with the expanded method introduce the formal written method, involving decomposition/exchange:

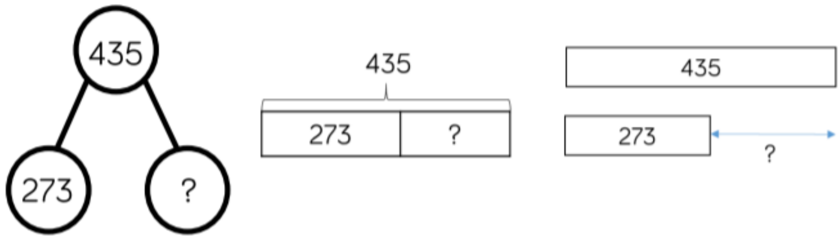
73 − 27 = 46

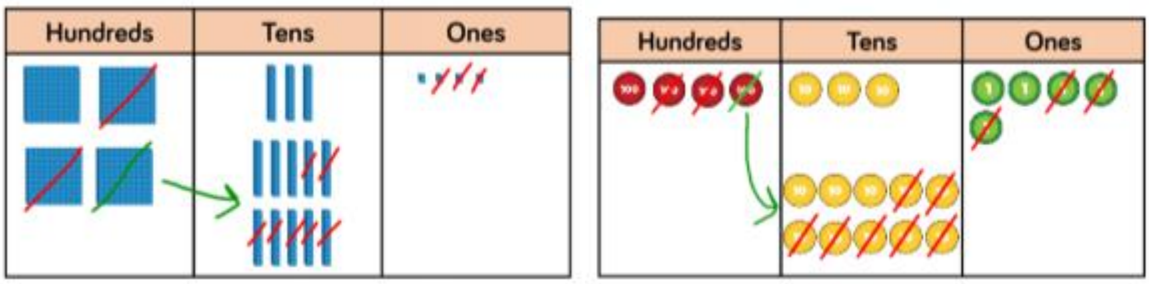
 6 13
  7 3
- 2 7
  4 6

Use the language of place value to ensure understanding.
'We can't subtract seven from three, so we need to exchange a ten for ten ones to give us 60 + 13.'

Use base ten materials to support understanding.

If children are confident, extend the use of the formal written method with numbers over 100, returning to the expanded method first, if necessary. In each instance consider the visual representations and concrete materials you will use to support understanding.





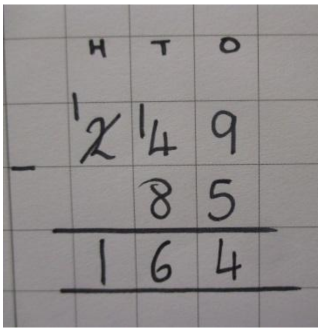
Subtraction - Year Four

- Subtract numbers with up to 4 digits using the formal written method of columnar subtraction where appropriate

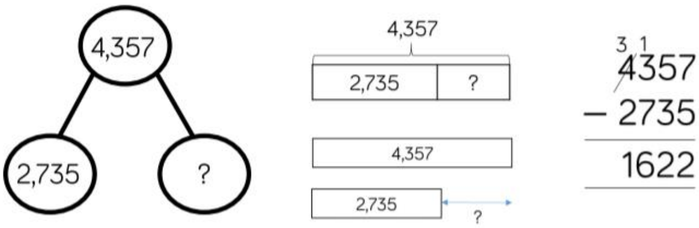
Continue to teach the use of empty number lines with three and four digit numbers, as appropriate.

Continue to develop the formal written method of subtraction by revisiting the expanded method first, if necessary (see Year 3). Continue to use visual representation and concrete materials to support understanding.

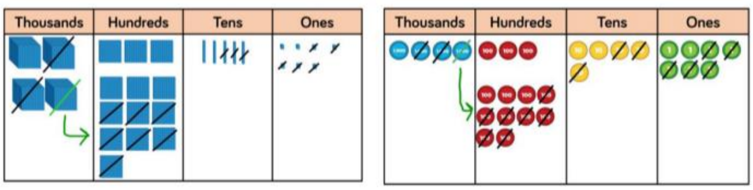
This will then lead onto a contracted formal written method:



When children are confident, develop with four digit numbers and decimal numbers (in the context of money and measures).



$$4{,}357 - 2{,}735 = 1{,}622$$



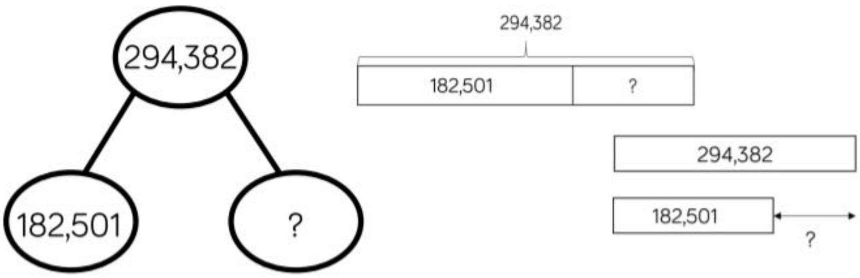
Subtraction - Year Five & Six

- Subtract whole numbers with more than 4 digits, including using formal written method (columnar subtraction)

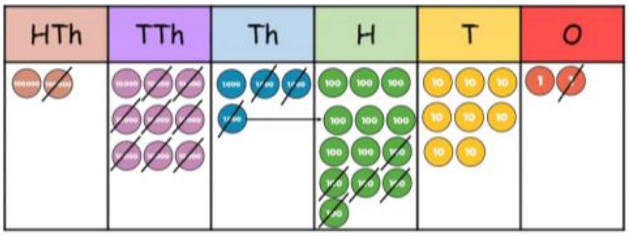
Continue to teach the use of empty number lines with larger numbers and decimals, as appropriate.

Continue to develop the formal written method for subtraction with three and four digit numbers (see Y4 guidance), returning to an expanded method and using base ten materials/place value counters, if necessary.

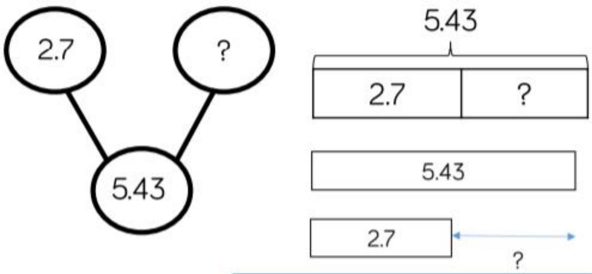
When children are confident extend with larger numbers (and decimal numbers). Return to an expanded method, if necessary.



294,382 – 182,501 = 111,881

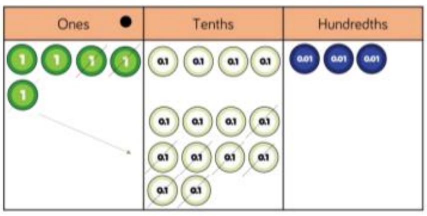


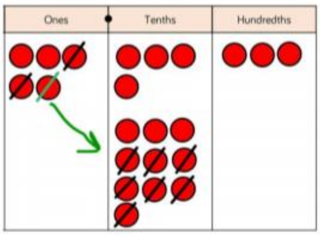
	2	9	~~4~~ 3	13	8	2
−	1	8	2	5	0	1
	1	1	1	8	8	1



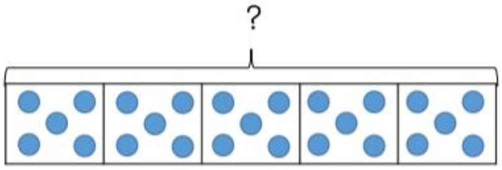
~~5~~ (4) .(1)43
+ 2.7
2.73

5.43 – 2.7 = 2.73

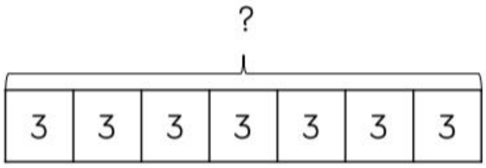




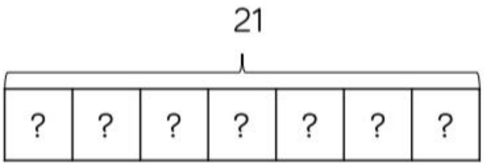
Bar Models



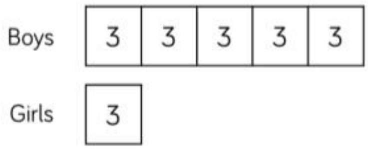
$5 \times 5 = 25$



$3 \times 7 = 21$
$7 \times 3 = 21$



$21 \div 7 = 3$



Benefits

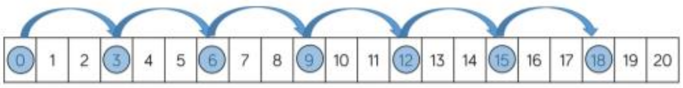
Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

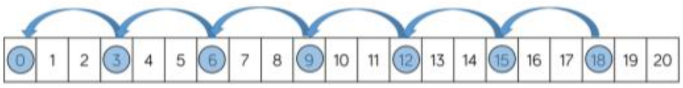
It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

Number Tracks



$6 \times 3 = 18$
$3 \times 6 = 18$



$18 \div 3 = 6$

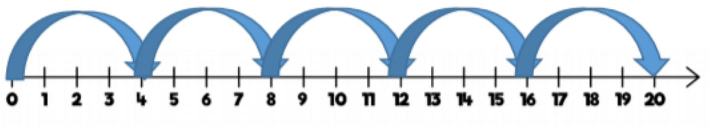
Benefits

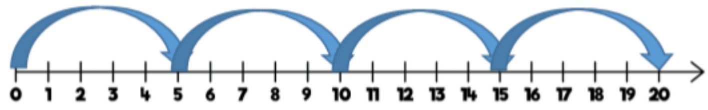
Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

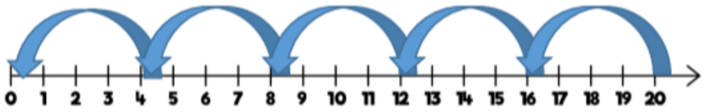
Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (Labelled)





$4 \times 5 = 20$
$5 \times 4 = 20$



$20 \div 4 = 5$

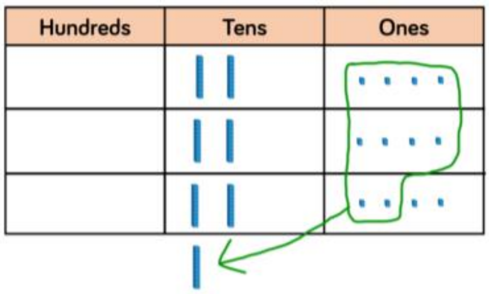
Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

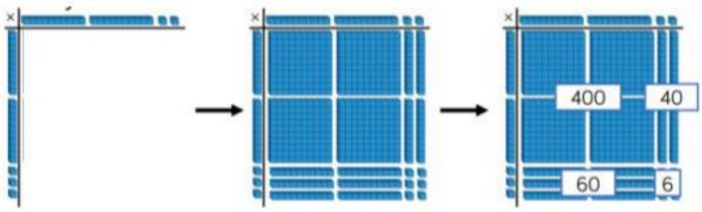
When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Base 10/Dienes (Multiplication)



$$
\begin{array}{r}
24 \\
\times \quad 3 \\
\hline
72 \\
\hline
1
\end{array}
$$

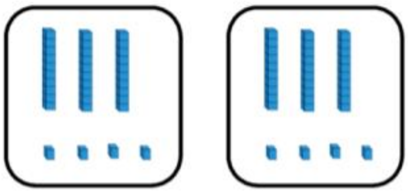


Benefits

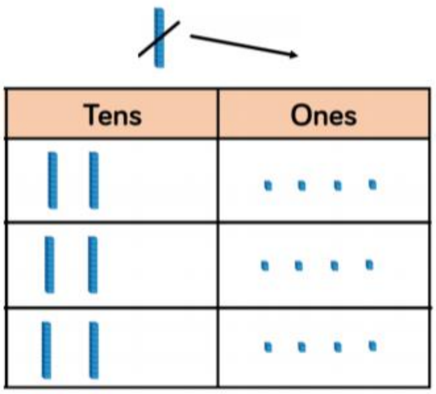
Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

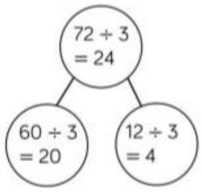
Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.



$68 \div 2 = 34$



$72 \div 3 = 24$



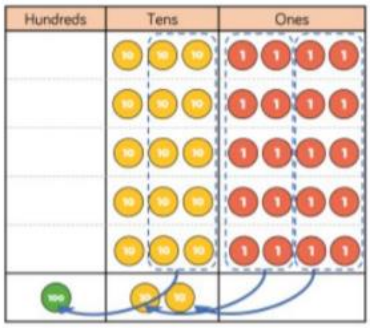
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

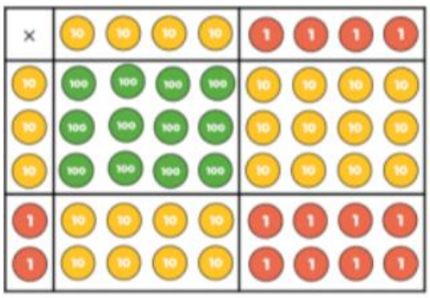
When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (Multiplication)




```
   34
 ×  5
  120
  1 2
```



```
    44
  × 32
     8
    80
   120
+ 1200
  1408
   1
```

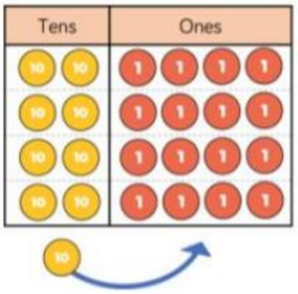
Benefits

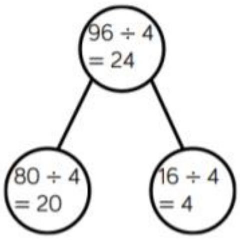
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

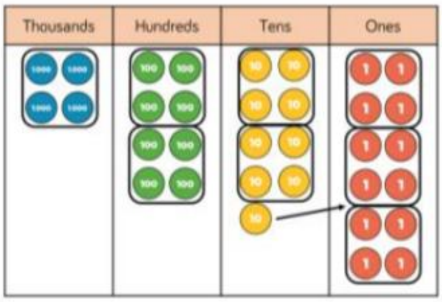
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (Division)







1223
4 | 4892

Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

Stages in Multiplication

Multiplication – EYFS

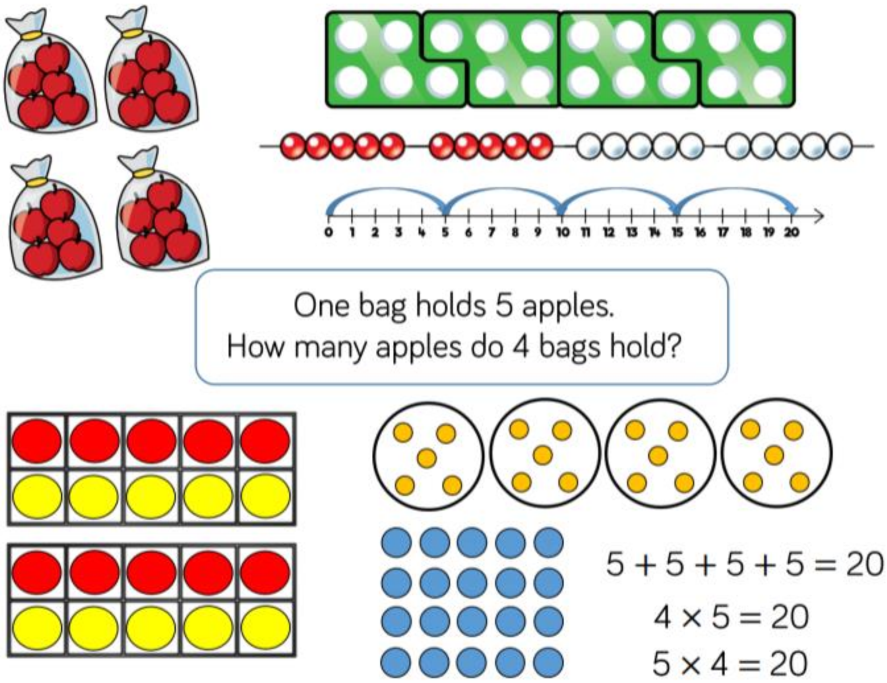
Children will engage in a wide variety of songs and rhymes, games and activities.
In practical activities and through discussion they will begin to solve problems involving doubling.

'Three apples for you and three apples for me. How many apples altogether?'

Multiplication - Year One

- Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Count in multiples of twos, fives and tens (to the 10th multiple)

Count in steps verbally and using visual representations including a number line. Start to use arrays to illustrate multiplication:



Multiplication - Year Two

- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables
- Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs
- Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts
- Show that multiplication of two numbers can be done in any order (commutative)

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the x sign to record.

Combine groups in repeated addition & ensure that children are familiar with seeing this represented as a bar model:

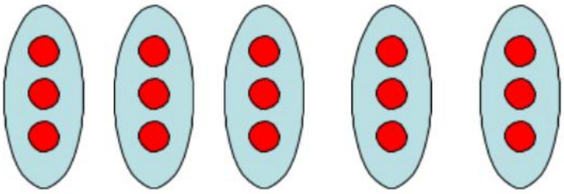
'3 groups of 10 crayons'

'How many crayons altogether?'

'10 + 10 + 10 = 30'

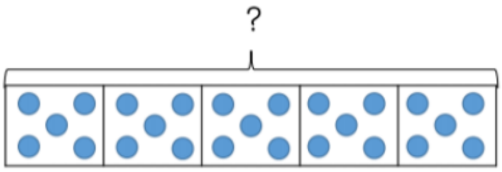
'3 groups of 10' '3 times ten'

'3 x 10 = 30' '10 x 3 = 30'

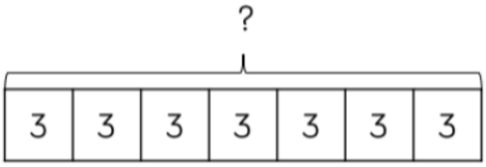


'5 groups of 3' '5 lots of 3' '3 + 3 + 3 + 3 +3 = 15'

'5 times 3' '3 multiplied by 5' '5 x 3 = 15' '3 x 5 = 15'



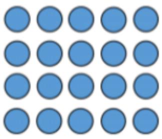
$5 \times 5 = 25$



$3 \times 7 = 21$

$7 \times 3 = 21$

Use arrays to support understanding:



$5 + 5 + 5 + 5 = 20$

$4 \times 5 = 20$

$5 \times 4 = 20$

<u>Multiplication - Year 3</u>

- Recall and use multiplication facts for the 3, 4 and 8 multiplication tables (continue to practise the 2, 5 and 10 multiplication tables)
- Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, <u>using mental</u> and progressing to a formal written method

Continue to use bar models, number lines and arrays to represent multiplication.

Partitioning method for multiplication of a two-digit number by a one-digit number:

13 x 5 = 65 (Partition 13 into 10 + 3)
10 x 5 = 50
3 x 5 = 15

50 + 15 = 65

Grid Method (two-digit number multiplied by a one- digit number):

13 x 8 = 10

X	10	3
8	80	24

80 + 24 = 104

'Partition 13 into 10 + 3 then multiply each number by 8. Add the partial products (80 and 24) together.'

This will lead into expanded short multiplication:

13 x 8 = 104

10 + 3 x 8

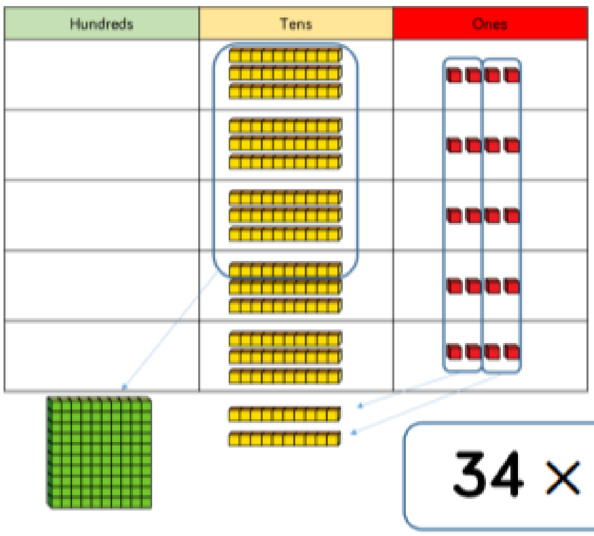
2 4 (3 x 8)

+ 8 0 (10 x 8)

1 0 4

Include an addition symbol when adding partial products.

Refine the recording in preparation for formal short multiplication:

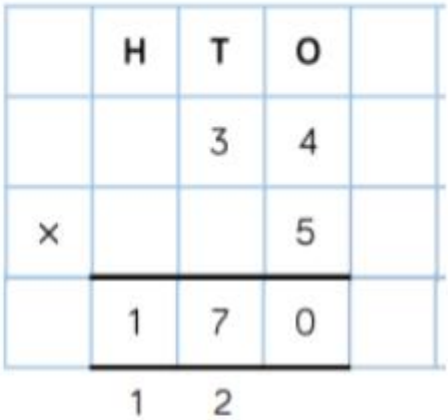


	H	T	O		
		3	4		
×			5		
		2	0	(5 × 4)	
+	1	5	0	(5 × 30)	
	1	7	0		

34 × 5 = 170

Ensure that this is modelled and taught alongside visual representations.

When children are confident, move onto formal short multiplication:



<u>Multiplication - Year Four</u>

- Recall multiplication facts for multiplication tables up to 12 × 12
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Further develop the grid method for two-digit numbers multiplied by a one- digit number.

36 x 4 = 144

X	30	6
4	120	24

120 + 24 = 144 (add the partial products)

Continue to practise the formal method of short multiplication of a two-digit number by a one -digit number throughout Y4.

If children are confident, continue to develop short multiplication with three- digit numbers multiplied by a one-digit number.

If necessary, return to the grid method and/or expanded method first:

127 x 6 = 762

x	100	20	7
6	600	120	42

600 + 120 + 42 = 762 (add the partial products)

This leads to expanded short multiplication:

127 x 6 = 762

```
   1 2 7
 x     6
     4 2   (6x7)
 + 1 2 0   (6x20)
   6 0 0   (6x100)
   7 6 2
```

This will lead into short multiplication (formal method):

	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

Multiplication - Year 5

- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

Build on the work covered in Y4 with the formal method of short multiplication (two-digit number multiplied by a one-digit number)

When children are confident, introduce multiplication by a two-digit number. If necessary, return to the grid method and/or expanded method first.

Grid method (two-digit number multiplied by a teen- number):

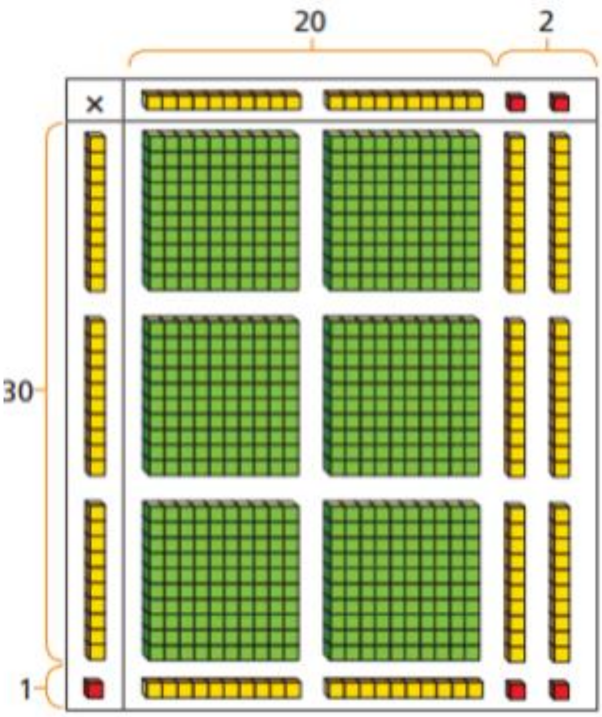
23 x 13 = (20 + 3) x (10 + 3) = 299

X	20	3
10	200	30
3	60	9

```
   2 3 0
 +   6 9
   2 9 9
```

Add the partial products (200 + 30) + (60 + 9) = 299

Grid method can be represented using Base 10/Dienes and/or Place Value Counters to aid understanding.



Expanded long multiplication (two-digit numbers multiplied by a teen- number):

23 x 13 = 299

  2 3
X 1 3
    9 (3 x 3)
  6 0 (3 x 20)
+ 3 0 (10 x 3)
2 0 0 (10 x 20)
2 9 9

This leads into...

Compact long multiplication (formal method):

23 x 13 = 299

  2 3
X 1 3
+ 6 9 (3 x 23)
2 3 0 (10 x 23)
2 9 9

Use the language of place value to ensure understanding.

Add the partial products.

Extend to larger two-digit numbers:

56 x 27 = (50 + 6) x (20 + 7) = 1512

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

Add the partial products (1000+ 120) + (350 + 42) = 1512

Expanded long multiplication (two-digit numbers multiplied by two-digit numbers):

```
    5 6
  X 2 7
    4 2   (7x6)
  3 5 0   (7x50)
+ 1 2 0   (20x6)
1 0 0 0   (20x50)
1 5 1 2
```

This leads into…

Compact long multiplication (formal method):

56 x 27 = 1512

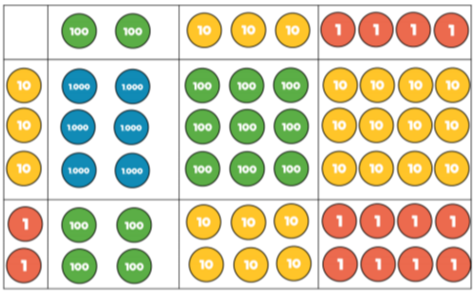
```
    5 6
  x 2 7
  3 9⁴2
+ 1 1¹2 0
  1 5 1 2
     1
```

Use the language of place value to ensure understanding.

In this example there are digits that have been ‘carried’ over in the partial products.

Add the partial products.

When children are confident with long multiplication extend with three-digit numbers multiplied by a two-digit number, returning to the grid method first, if necessary:



Th	H	T	O
	2	3	4
×		3	2
	4	6	8
$_1$7	$_1$0	2	0
7	4	8	8

×	200	30	4
30	6,000	900	120
2	400	60	8

234 × 32 = 7,488

Extend with short and long multiplication of decimal numbers (initially in the context of money and measures), returning to an expanded method first, if necessary (see Y6 guidance).

<u>Multiplication - Year 6</u>

- Multiply multi-digit numbers (including decimals) up to 4 digits by a two- digit whole numbers
-

Continue to practise and develop the formal short multiplication method and formal long multiplication method with larger numbers and decimals throughout Y6. Return to an expanded forms of calculation initially, if necessary (see Y5 guidance).

Four digits by two digits:

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 $_2$	1 $_5$	9 $_3$	1 $_7$	2
5 $_1$	4	7 $_1$	8	0
7	6	6	9	2
		1		

2,739 × 28 = 76,692

The grid method (decimal number multiplied by a two-digit number):

53·2 x 24 = 1276·8

X	50	3	0·2	
20	1000	60	4	1064·0
4	200	12	0·8	212·8
				1276·8

The formal written method of long multiplication:

5 3·2		
x 2 4·0		
2 1^{1}2 ·8	(53·2 x 4)	
1 0 6 4 ·0	(53·2 x 20)	
1 2 7 6 ·8		

It is an option to include ·0 in this example, but not essential.

The prompts (in brackets) can be omitted if children no longer need them.

Stages in Division

Division - EYFS

Children will engage in a wide variety of songs and rhymes, games and activities.
In practical activities and through discussion they will begin to solve problems involving halving and sharing.


Share the apples between two people.

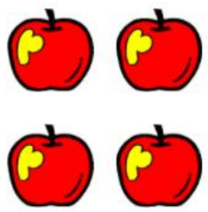
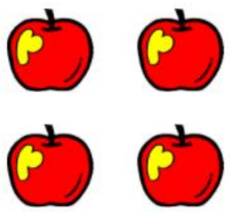
‘Half of the apples for you and half of the apples for me.’

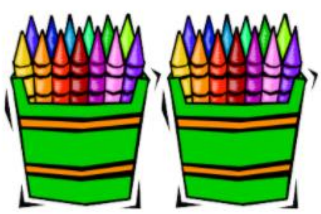
Division - Year One

- Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Count in multiples of twos, fives and tens (to the 10th multiple)

Children will start with practical sharing using a variety of resources. They will share objects into equal groups in a variety of situations.
They will begin to use the vocabulary associated with division in practical contexts.

‘Share these eight apples equally between two children. How many apples will each child have?’

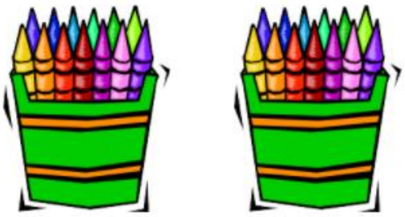





‘Share 20 crayons between 2 pots.’

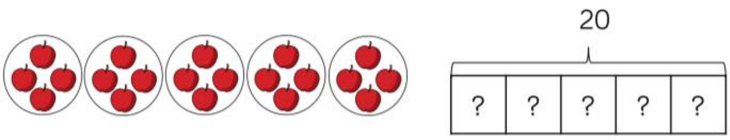
‘How many crayons are in each pot?’

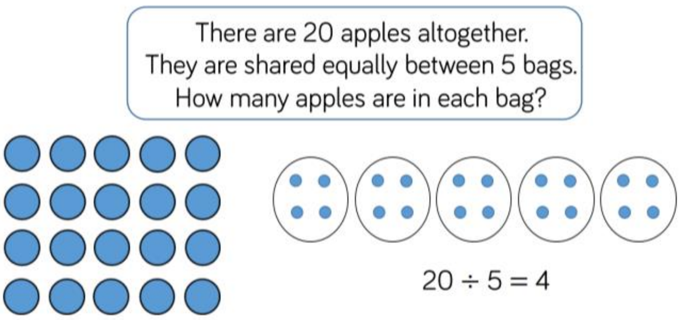
Children will move from sharing to grouping in a practical way:



‘Put 20 crayons into groups of 10. How many pots do we need?’

Move onto sharing representations including arrays to support early division (emphasise links to multiplication):





<u>Division - Year Two</u>

- Recall and use division facts for the 2, 5 and 10 multiplication tables
- Calculate mathematical statements for division within the multiplication tables they know and write them using the division (÷) and equals (=) signs
- Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts

Children will use a range of vocabulary to describe division and use practical resources, pictures, diagrams and the ÷sign to record, using multiples that they know.

Sharing and grouping:

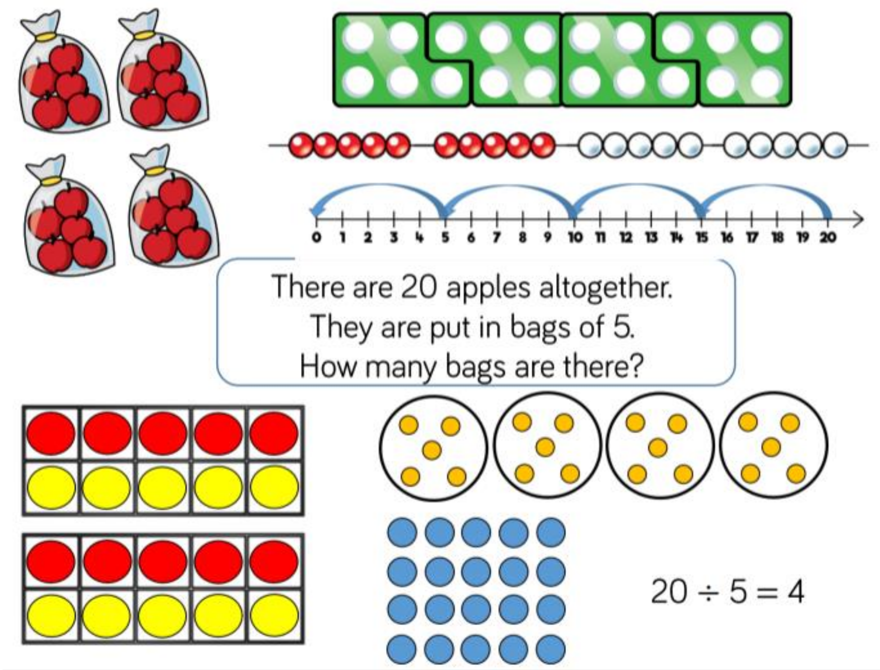
‘30 crayons shared equally between three pots.’ (Sharing)
‘We have 30 crayons and put ten crayons in each pot.
How many pots do we need?’ (Grouping)

‘30 divided by 10 = 3’
‘30 divided by 3= 10’

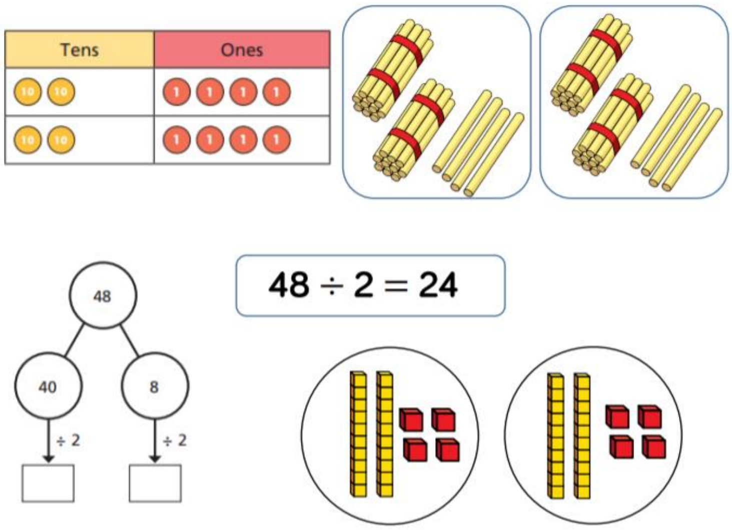
30 ÷ 10 = 3

30 ÷ 3 = 10

Represent grouping problems using and use terminology of e.g. 20 ‘shared into groups of 5’



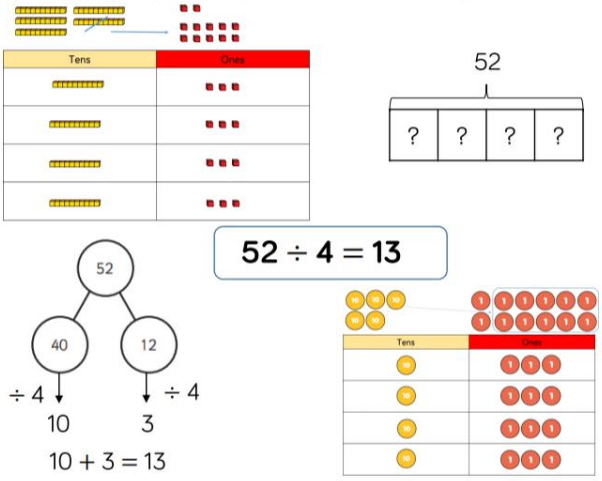
Once children are confident, use concrete apparatus to model division by partitioning two-digit numbers with no exchange.



<u>Division - Year 3</u>

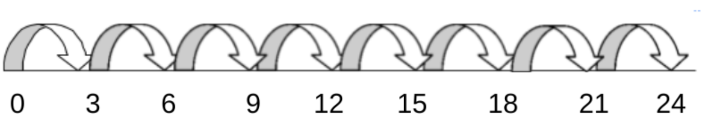
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables (continue to practise the 2, 5 and 10 multiplication tables)
- Write and calculate mathematical statements for division using the multiplication tables that they know using mental and progressing to a formal written method
- Use known times tables facts to divide multiples of 10. E.g. 60 ÷ 3 using the formal written method

Continue to use practical resources, pictures, diagrams, number lines, arrays and the ÷ sign to record, using multiples that they know, as appropriate (see Y2 guidance).



Use an empty number line to count forwards:

24 ÷ 3 = 8



'How many threes in 24?'

Also count backwards from 24 in 3's to make the link with repeated subtraction.

Introduce the formal layout for division using multiplication/division facts that children know. Refer to this as <u>Bus Stop Method</u>.

24 ÷ 3 = 8

This can also be recorded as…

```
     08
   ______
3 ) 24
```

'Twenty four divided by three equals eight.'

'How many threes are there in twenty four?'

Move onto using known facts to divide multiples of ten:

```
     20
   ______
3 ) 60
```

Division - Year Four

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Use place value, known and derived facts to divide mentally
- Divide two-digit and three-digit numbers by a one-digit number using formal written layout (not explicitly stated in the programmes of study but implied in the non-statutory guidance)

Continue to write and calculate mathematical statements for division using the multiplication tables that the children know e.g.

32 ÷ 8 = 4

Continue using the formal written layout for division using multiplication tables that they know:

$$8\,\overline{)\;32}\quad \text{answer: } 04$$

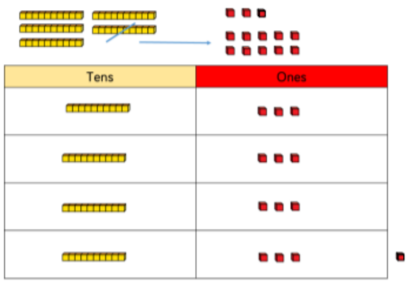
‘How many eights are there in thirty two?’

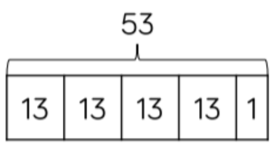
Continue using the formal written layout, introducing remainders:

25 ÷ 3 = 8 r1

$$3\,\overline{)\;25}\quad \text{answer: } 08 \text{ r } 1$$

NB Remainders are not specifically referred to until Y5 in the National Curriculum. However, this may be an appropriate point to introduce them using familiar multiplication facts if children are confident using these methods. Model this using concrete and pictorial representations.





Division using partitioning (two digits divided by one digit):

65 ÷ 5 = 13

65 = 50 + 15 Partition 65 into 50 and 15

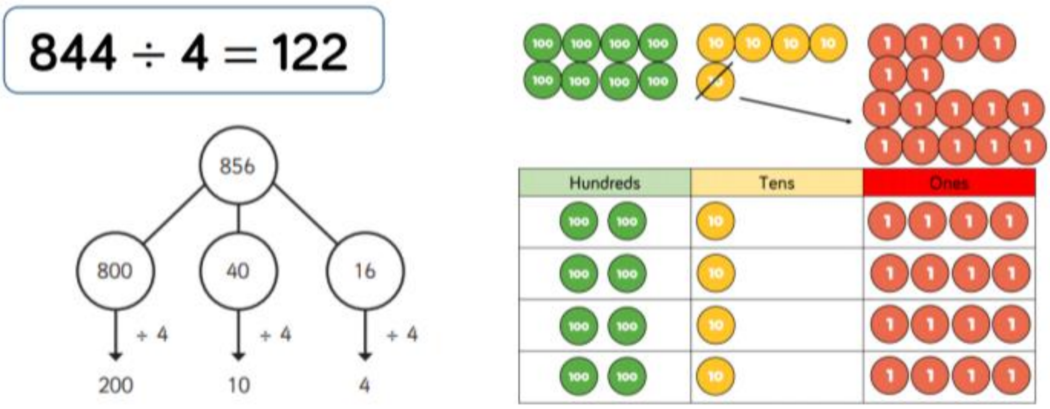
50 ÷ 5 = 10

15 ÷ 5 = 3

10 + 3 = 13

NB Children will need to practise partitioning in a variety of ways.

Move onto dividing three-digit numbers by a single-digit number using partitioning. These should be divisible without a remainder.



This will lead into the formal written method of short division:

98 ÷ 7 = 14

  1 4
7) 9 ^{2}8

Use the vocabulary of place value to ensure understanding and make the link to partitioning.

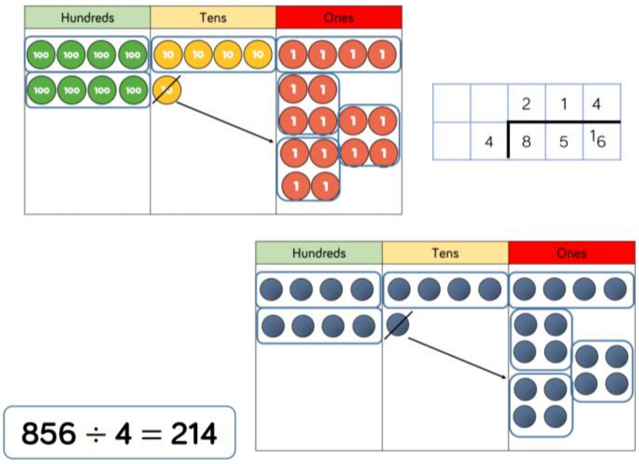
Continue to practise the formal method of short division throughout Y4.

If children are confident develop further, by dividing three-digit numbers by a one- digit number using the formal method of short division with whole number answers (no remainders).

<u>Division - Year Five</u>

- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Continue to practise the formal written method of short division with whole number answers…



...and with remainers

$432 \div 5 = 86 \text{ r}2$

```
     0 8 6 r 2
5 ) 4 ⁴3 ³2
```

The remainder can also be expressed as a fraction, $\frac{2}{5}$ (the remainder divided by the divisor): $432 \div 5 = 86\frac{2}{5}$

Continue to practise, develop and extend the formal method of short division, with and without remainders. Interpret and express remainders according to the context.

Division - Year Six

- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions/decimals, or by rounding, as appropriate for the context

Continue to practise the formal method of short division, with and without remainders, using the language of place value to ensure understanding (see Y5 guidance).

		0	3	6
	12	4	$^{4}3$	$^{7}2$

$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$

	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

15	30	45	60	75	90	105	120	135	150

The remainder can also be expressed as a fraction (the remainder divided by the divisor).

Dividing by a two-digit number using a formal method of long division (without a remainder):

		0	3	6	
1	2	4	3	2	(×30)
	−	3	6	0	
			7	2	(×6)
	−		7	2	
				0	

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$

	0	4	8	9	
15	7	3	3	5	
−	6	0	0	0	(×400
	1	3	3	5	
−	1	2	0	0	(×80)
		1	3	5	
−		1	3	5	(×9)
				0	

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

Dividing by a two-digit number using a formal method of long division (with a remainder):

$372 \div 15 = 24 \text{ r}12$

			2	4	r	1	2
1	5	3	7	2			
	−	3	0	0			
			7	2			
	−		6	0			
			1	2			

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

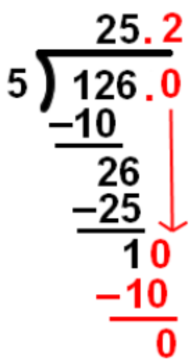
			2	4	$\frac{4}{5}$
1	5	3	7	2	
	−	3	0	0	
			7	2	
	−		6	0	
			1	2	

$372 \div 15 = 24\frac{4}{5}$

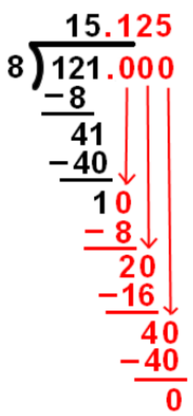
Formal method of long division, expressing a remainder as a decimal.

NB Only teach this method when children are completely secure with the previous method.

One decimal place...



More than one decimal place...



Glossary

Addition & Subtraction

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total – The aggregate or the sum found by addition.

Multiplication & Division

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor

---